

Suppression and Revival of Weak Localization through Control of Time-Reversal Symmetry

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We report on the observation of suppression and revival of coherent backscattering of ultra-cold atoms launched in an optical disorder and submitted to a short dephasing pulse, as proposed in a recent paper of T. Micklitz *et al.* [arXiv:1406.6915]. This observation, in a quasi-2D geometry, demonstrates a novel and general method to study weak localization by manipulating time reversal symmetry in disordered systems. In future experiments, this scheme could be extended to investigate higher order localization processes at the heart of Anderson (strong) localization.

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Weak localization (WL), a precursor of Anderson localization [1], is a fundamental paradigm in mesoscopic physics [2]. It arises from constructive interferences between *time-reversed* multiple-scattering paths. These interferences increase the probability that an electron remains around its initial position, thereby acting against propagation and resulting in an increased resistivity [3, 4]. WL (and thus the resistivity) can be modified if time reversal symmetry (TRS) is perturbed or broken in the system [5]. An elegant method is the application of an external magnetic field that introduces a controlled dephasing between the counter-propagating paths. Historically, WL was first introduced to interpret negative magnetoresistance effects in thin metallic films [6], and was unambiguously demonstrated by the seminal observation of the oscillation of the resistivity in a thin walled cylinder in the presence of an external magnetic field [7]. In the same spirit, other methods have been used to study WL in electronic systems by controlled TRS breaking, either by using magnetic impurities and spin-orbit coupling [6, 8] or by application of time dependent potentials, e.g., using high frequency RF fields [9, 10]. In the latter case, phase decoherence entails an irreversible suppression of WL, in contrast to the experiment using magnetic fields where the coherence is preserved.

In AMO physics, the equivalent of weak localization is the phenomenon of coherent back scattering (CBS), leading to an enhancement of the scattering probability in the backward direction (ideally by a factor 2) when a plane wave is launched in a disordered scattering medium (see Fig. 1a). It has been observed with a variety of classical waves [2], including optical, acoustic, elastic and seismic waves. It has recently been observed with ultracold atoms [11, 12], following the proposal in Ref. [13]. In direct analogy with the experiments conducted in condensed matter physics, the influence of TRS breaking on CBS has been studied, for instance using acoustic waves in a rotating medium, which simulates a mag-

netic field [14]. For light various mechanisms that break TRS have been considered (see, e.g., [15]). On the one hand the decay of the CBS signal due to pure dephasing processes linked to the depolarization in a Faraday medium submitted to a magnetic field [16, 17], or to internal structures in cold atomic samples [18], has been observed. On the other hand, the irreversible suppression of CBS by decoherence either due to the motion of the scatterers [19], or to an ultrafast modulation of the disordered medium [20], has been investigated.

In this Letter, we report on the demonstration of a novel and general method, proposed by T. Micklitz *et*

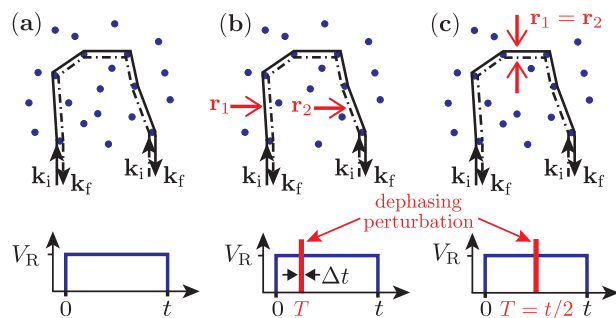


FIG. 1. Principle of CBS suppression and revival in a 2D experiment with ultracold atoms. (a) CBS results from the constructive interference between the scattering amplitudes of a direct (1, solid line) and reciprocal (2, dashed) multiple scattering path, for a plane matter wave launched with a wave vector \mathbf{k}_i and detected in the backward direction at $\mathbf{k}_f = -\mathbf{k}_i$. The time diagram shows the switching on and off of the disordered potential V_R . (b) CBS suppression: A pulsed perturbation at time $T \neq t/2$ entails a phase difference $[\phi_{\text{kick}}(\mathbf{r}_1) \neq \phi_{\text{kick}}(\mathbf{r}_2)]$ between the amplitudes associated with paths 1 and 2, where \mathbf{r}_1 and \mathbf{r}_2 are the positions on each path at time T . The constructive interference is destroyed. (c) CBS revival: For the special case $T = t/2$, one has $\mathbf{r}_1 = \mathbf{r}_2$ and the constructive interference between the direct and reciprocal paths is restored.

al. [21], to study WL by TRS breaking, without destroying phase coherence. Our experimental system consists of ultracold atoms propagating in an optical disordered potential, in a quasi two-dimensional (2D) configuration, for which CBS can be observed as a peak in the momentum distribution as described in [11]. In the present experiment, TRS is broken by a short dephasing kick applied at time T . After this kick we observe the suppression of the CBS peak, followed by a brief revival at time $2T$. This revival constitutes a new and direct signature of phase coherence in disordered media. It also strikingly highlights the role of time reversal symmetry in WL [21].

The principle of the suppression and revival effect is illustrated in Fig. 1. Consider atoms with initial momentum $\hbar\mathbf{k}_i$ that are launched into a disordered potential at time $t = 0$. In the absence of any perturbation the constructive interference between counter propagating multiple scattering paths leads to an enhancement of the scattering probability at $\mathbf{k}_f = -\mathbf{k}_i$ [Fig. 1(a)], i.e., the CBS peak. At time $t = T$ we now apply, for a short duration Δt , a magnetic field gradient that imposes a linear potential $V(\mathbf{r})$ on the atoms. This entails a momentum change $\hbar\Delta\mathbf{k} = -\nabla V(\mathbf{r}) \Delta t$, associated with a supplementary *local* phase shift $\phi_{\text{kick}}(\mathbf{r}) = V(\mathbf{r})\Delta t/\hbar$ that depends on the position \mathbf{r} of the atom on the scattering path at time T [22]. This results in a dephasing between the counter-propagating paths,

$$\Delta\phi_{\text{kick}} = \phi_{\text{kick}}(\mathbf{r}_2) - \phi_{\text{kick}}(\mathbf{r}_1) = \Delta\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2), \quad (1)$$

where $\mathbf{r}_{1,2}$ correspond to the respective positions on each path [Fig. 1(b)]. Hence the CBS peak is *a priori* suppressed for observation times $t > T$. If, however, the observation time is chosen at $t = 2T$, the two counter-propagating paths acquire the same extra phase [Fig. 1(c)], and the constructive interference is preserved. In other words, the TRS, and consequently the CBS

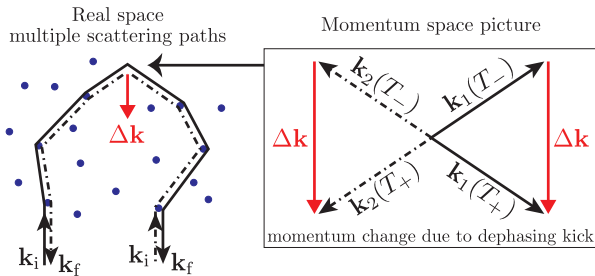


FIG. 2. **Time-reversed multiple scattering paths in the case of a strong kick.** The direct path (1, solid line), which includes the \mathbf{k} -vector change due to the kick at $t = T$ (shown in the inset), has a \mathbf{k} -vector $\mathbf{k}_f = -\mathbf{k}_i$ at $t = 2T$. The counter-propagating path (2, dashed line), which includes the same \mathbf{k} -vector change at $t = T$ (see inset), is time-reversal symmetric. Thus, the two paths interfere constructively, leading to the CBS revival at $2T$ around $\mathbf{k}_f = -\mathbf{k}_i$.

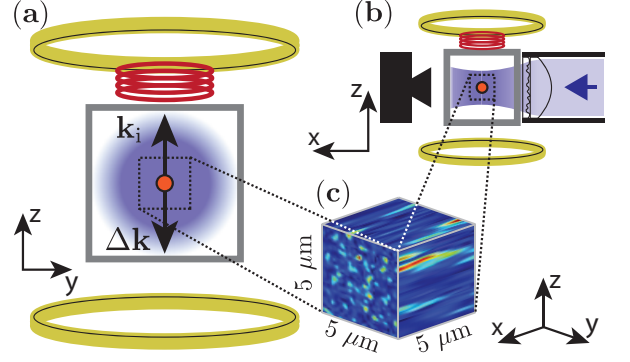


FIG. 3. **Experimental Setup.** (a) and (b): A cloud of ultracold atoms (orange disk) is launched upwards, along the z axis, with an initial momentum $\hbar\mathbf{k}_i$. The disordered potential is created by an anisotropic laser speckle, elongated along the x axis, obtained by passing a laser beam through a scattering plate (shaded blue). The atoms are suspended against gravity by magnetic levitation provided by the pair of large coils (yellow). The small red coil is used to create a magnetic gradient pulse resulting in a downward momentum kick $\hbar\Delta\mathbf{k}$, along the z axis, to the atoms. After a time of flight of 150 ms, fluorescence imaging recorded by an EMCCD camera along x gives the transverse momentum distribution (in the $y-z$ plane, see text). (c) 3D false color representation of the disordered potential.

peak, are restored. Altogether, we expect a suppression of the CBS for $T < t < 2T$ and $t > 2T$, with a revival at $t = 2T$.

The reasoning above is valid in the perturbative regime considered in [21], where the kick is small enough that one can neglect the subsequent modifications of the scattering paths. Actually, the argument still holds for a large kick, as is the case in the experiment. A CBS revival is indeed expected at $\mathbf{k}_f = -\mathbf{k}_i$, whatever the kick's strength, due to the following reason: monitoring the momentum distribution at that exact backward value *automatically selects* those scattering paths for which energy is conserved after the kick (scattering from the disordered potential is considered elastic). As illustrated in Fig. 2, these paths satisfy $|\mathbf{k}(T_+)| = |\mathbf{k}(T_-) + \Delta\mathbf{k}| = |\mathbf{k}(T_-)|$, where T_- and T_+ are the times just before and after the kick [23]. When the observation is done at $t = 2T$, each such scattering path has an exact counter-propagating one, even though they are strongly modified by the kick at time T . Here again, TRS is restored and a CBS revival is expected with a contrast ideally equal to one.

Our experimental set-up, sketched in Fig. 3, has been described in [11]. A salient feature is the ability to launch a “quasi-monochromatic” cloud of non-interacting atoms into the disorder. The cloud, containing 10^5 ^{87}Rb atoms prepared in the ground hyperfine Zeeman sub-level $|F = 2, m_F = -2\rangle$, is suspended against gravity by a vertical magnetic field gradient. It is launched along z with a mean velocity of $v_i = 3.09 \pm 0.04$ mm/s ($k_i = 4.24 \mu\text{m}^{-1}$,

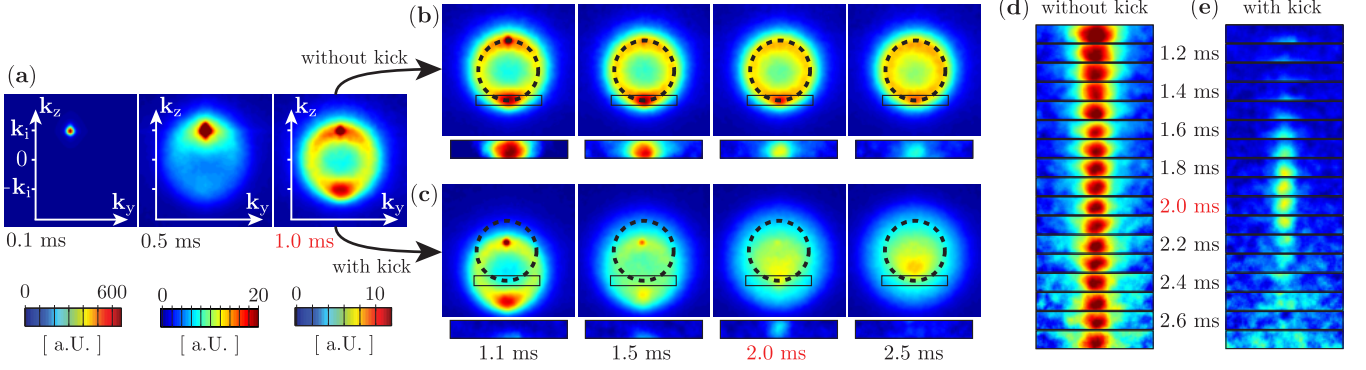


FIG. 4. **Observation of the CBS revival.** The images represent the 2D \mathbf{k} -vector distribution in the $y - z$ plane, $n(\mathbf{k}, t)$. Each image results from an average over 30 experimental runs and the color scale is kept unchanged, except for the first two images in (a). (a) Initial evolution of $n(\mathbf{k}, t)$ for atoms launched at $t = 0$ in the disorder with momentum $\hbar \mathbf{k}_i$. The initial peak at \mathbf{k}_i decays while the atoms \mathbf{k} -vectors are redistributed on a ring of radius $|\mathbf{k}_i|$, and the CBS peak grows at $-\mathbf{k}_i$. (b) and (c): Evolution for $t > T$, without and with the dephasing kick at $T = 1$ ms. The dashed circles are centered on the origin, and have a radius $|\mathbf{k}_i|$. The rectangular boxes below each image show the extracted coherent fraction $C_{\text{coh}}(\mathbf{k}, t)$ around $-\mathbf{k}_i$ (see text). (d) and (e) Coherent fractions around $-\mathbf{k}_i$, as in (b) and (c), but renormalized by the reference CBS peak at the same time (see text).

where $\mathbf{k} = m\mathbf{v}/\hbar$ and m the atom mass), and a very narrow velocity spread of 0.18 ± 0.02 mm/s. At $t = 0$, we switch on an anisotropic speckle field, elongated along the x axis, created by a far off resonance laser (wavelength 532 nm) [24, 25]. The disordered potential has correlation lengths much shorter in the y - z plane than in the x direction ($\sigma_y = \sigma_z = 0.27 \mu\text{m}$ and $\sigma_x = 1.40 \mu\text{m}$ HWHM), so that the situation can be considered 2D for atoms launched in the y - z plane [11]. At time t , the disorder is switched off and the 2D atomic velocity distribution [or equivalently the \mathbf{k} -vector distribution $n(\mathbf{k}, t)$ with \mathbf{k} in the y - z plane] is recorded by fluorescence imaging, along the x axis, after a free expansion of 150 ms. Taking into account the initial size of the cloud, the overall resolution is estimated to be $\Delta k_{\text{res}} = 0.3 \mu\text{m}^{-1}$.

The dephasing kick is realized by a pulsed magnetic field with a gradient of about 100 G/cm along the z axis (Fig. 3). We vary the pulse duration Δt to adjust the strength Δk of the kick, which is directly measured in the experiment (see below). This duration is always less than 100 μs , i.e., shorter than the mean scattering time.

Figure 4 shows the evolution of the \mathbf{k} -vector distribution when atoms are subjected to the disordered potential, with or without the dephasing kick applied. The average disorder amplitude is set to $V_R/\hbar = 660$ Hz [26]. Panels (a) and (b) correspond to the reference case, i.e., when no kick is applied ($\Delta \mathbf{k} = 0$). In the beginning, the initial narrow distribution centered around \mathbf{k}_i decays as the momenta are redistributed on a ring of mean radius $|\mathbf{k}_i|$ (elastic scattering). Monitoring that decay and the isotropization of the momentum distribution, we infer the mean scattering time $\tau_s = 0.22$ ms (hence $l_s = v_i \tau_s \sim 0.7 \mu\text{m}$) and the transport time $\tau^* = 0.6$ ms (hence $l^* = v_i \tau^* \sim 1.8 \mu\text{m}$) [27]. After a few scattering

events the CBS peak develops around $\mathbf{k}_f = -\mathbf{k}_i$ and becomes clearly visible at $t = 1$ ms. As explained in [11], we observe a decrease of the contrast of the CBS peak for longer times, which can be attributed to two reasons. Firstly, the CBS peak width becomes smaller than the experimental resolution. Secondly, after a few scattering events the probability for the atom to scatter out of the y - z plane becomes significant. As a consequence, the observation time of the CBS peak dynamics is limited to about $6 \tau^*$ (3.5 ms) in our experiment.

Panel (c) of Fig. 4 shows the evolution of the momentum distribution when a dephasing kick is applied along z at time $T = 1$ ms (kick duration $\Delta t = 35 \mu\text{s}$ FWHM). This kick entails a momentum change of $\hbar \Delta \mathbf{k}$ in all atoms: the whole \mathbf{k} -vector distribution, including the CBS peak, is thus shifted downwards (first image) by $\Delta k = -3.44 \pm 0.3 \mu\text{m}^{-1}$ ($\Delta k \sim -0.8 k_i$). The momenta are then redistributed by elastic scattering, and, after a few τ^* , the momentum distribution tends towards a broad, isotropic distribution. During that process, the CBS peak is rapidly suppressed. This is because of the large value of the kick amplitude Δk , chosen to ensure a rapid dephasing (to be more precise, after one scattering time $\tau_s = 0.22$ ms, the dephasing can be as large as $\Delta \phi_{\text{kick}} \sim \Delta k l_s = 2.3 > 1$ [see Eq. (1)]).

The revival of the CBS peak is expected to appear at $t = 2T = 2$ ms around $\mathbf{k}_f = -\mathbf{k}_i$, on top of an incoherent background. In order to reveal it, we first estimate the incoherent background $n_{\text{incoh}}(\mathbf{k}, t)$ by performing a quadratic fit of the distribution outside a rectangular box centered on $-\mathbf{k}_i$ and further extrapolating it into that box. The coherent fraction, defined as $C_{\text{coh}}(\mathbf{k}, t) = [n(\mathbf{k}, t) - n_{\text{incoh}}(\mathbf{k}, t)]/n_{\text{incoh}}(\mathbf{k}, t)$, is then extracted. Panel (e) shows that coherent fraction after

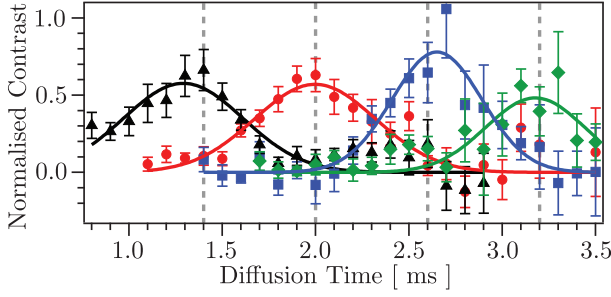


FIG. 5. **Amplitude of the CBS revivals** $\gamma_{\text{rev}}(t)$ for several dephasing times T . The data points \blacktriangle \bullet \blacksquare \blacklozenge correspond to $T = 0.7, 1.0, 1.3$, and 1.6 ms respectively, the kick's strength Δk being fixed (same as in Fig. 4). The dotted vertical lines indicate times $2T$ when the revivals are expected. Solid lines are Gaussian fits with zero offset. The observed revival times, determined by the fits, are respectively (in ms): $1.3 \pm 0.08, 1.99 \pm 0.08, 2.65 \pm 0.05$ and 3.18 ± 0.09 . Uncertainties correspond to the 95% confidence intervals.

application of the kick, hereafter referred to as $C_{\text{kick}}(\mathbf{k}, t)$ [for comparison the coherent fraction in the absence of the kick, $C_{\text{ref}}(\mathbf{k}, t)$, is shown in (d), and in both panels the coherent fractions are normalized by $C_{\text{ref}}(-\mathbf{k}_i, t)$]. A clear revival of the CBS peak is observed around $t = 2$ ms, a striking evidence of the predicted phenomenon.

Figure 5 shows the temporal evolution of the revival, whose amplitude is defined as $\gamma_{\text{rev}}(t) = C_{\text{kick}}(-\mathbf{k}_i, t)/C_{\text{ref}}(-\mathbf{k}_i, t)$, for several values of the kick time T and the same kick's strength Δk . In all cases, a revival is observed around the expected time, with a contrast of about 60%. We suspect that spurious magnetic fields, due to eddy currents that are excited by the strong magnetic dephasing pulse, are responsible for the reduced contrast of the revival. The revival shape is well fitted by a Gaussian with almost the same *rms* width for all cases, about $\Delta\tau_{\text{rev}} = 0.28 \pm 0.04$ ms.

In order to render an account of the shape and the width of the revival peak, we refer to the perturbative expression (1) of the phase difference between the amplitudes of counter-propagating scattering paths that contribute to the CBS signal at $\mathbf{k}_f = -\mathbf{k}_i$ (see Fig. 1). The amplitude of the revival is expected to vary as $\gamma(t) = \langle \exp[i\Delta\phi_{\text{kick}}(\mathbf{R})] \rangle$, where the brackets indicate a statistical average over the separation $\mathbf{R}(t) = \mathbf{r}_1 - \mathbf{r}_2$, for fixed T . This separation, null at $t = 2T$, is the distance corresponding to the propagation of the atoms in the disorder for a duration $|t - 2T|$.

If this propagation was a random walk, the distance $\mathbf{R}(t)$ would have a Gaussian distribution, with variance $\langle \mathbf{R}^2 \rangle = 2D|t - 2T|$, where $D = v l^*/2$ is the diffusion constant. The amplitude of the revival would then be

$$\gamma_{\text{dif}}(t) = e^{-\Delta k^2 \langle \mathbf{R}(t)^2 \rangle / 2} = e^{-\Delta k^2 D |t - 2T|}. \quad (2)$$

This formula is identical to the one derived by Micklitz *et al.* [21]. It predicts a profile with a symmetrical exponen-

tial shape, of half-width (at $1/e$) $\Delta\tau_{\text{dif}} = [D\Delta k^2]^{-1} \sim 0.03$ ms for our parameters. It does not correspond to our observation of a Gaussian shape (see Fig. 5), with a width one order of magnitude larger.

Actually, in our experiment, the diffusive hypothesis used in Eq. (2) is definitely not fulfilled at the short time scale characterizing the revival width, which is on the order of the mean scattering time τ_s . A similar failure of the diffusive hypothesis was already observed in [11] and we had rendered an excellent account of the observation at short times by using an effective ballistic dynamics, derived from [28], in which $\langle \mathbf{R}^2 \rangle = (v_i |t - 2T|/3)^2$. Using the same ansatz here, we obtain a Gaussian expression for the CBS revival profile:

$$\gamma_{\text{bal}}(t) = e^{-(t-2T)^2/2\Delta\tau_{\text{bal}}^2}, \text{ where } \Delta\tau_{\text{bal}} = \frac{3}{|\Delta k|v_i}. \quad (3)$$

For our parameters, one has $\Delta\tau_{\text{bal}} = 0.28$ ms, in striking agreement with the observations. Let us note that we have varied the strength of the kick Δk , and the revival widths were found in good agreement with expression (3) [29].

In conclusion, we have demonstrated experimentally a new method to break and restore TRS in a disordered medium, resulting in the suppression and revival of the CBS peak. While conceptually highlighting the profound role of TRS on CBS (and more generally WL), our observation also provides a novel indisputable signature of the role of coherence in weak localization. The method could serve to differentiate CBS from classical echoes as reported in [12]. It could be implemented differently, for instance kicking the disordered potential, or any kind of inhomogeneous external potential. It would be interesting to compare it to schemes using time dependent potentials [30], or artificial gauge fields [31]. Finally, extending the scheme to multiple kick sequences opens new prospects to study Anderson (strong) localization in a renewed perspective [21]. Depending on the chosen sequence, suppression and revivals of both the CBS and of the expected coherent forward scattering peak [32] could be observed and used to investigate higher-order mechanisms at the heart of Anderson localization. Such observations would complement ideally previous studies of Anderson localization with ultracold atoms [33, 34].

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